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MODAL PARAMETERS EXTRACTION USING WAVELETS

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Abstract. *This work proposes the use of wavelets in modal parameters identification of vibratory systems. The developed metodology uses optimization techniques as a "Matching Pursuit" algorithm modification. Signals obtained from simulation of a vibratory system with artificially added noise and from experimental tests are used to evaluate the method robustness, accuracy and reliability. The method performance is compared to the Ruzzene's method.*

Key Words: *Modal Analysis, Wavelets, Optimization.*

1. INTRODUCTION

The parameters of non stationary signals can only be computed in function of time, resulting in instantaneous values. (Bendat and Pierson, 1986)

The Short Time Fourier Transform, Wigner-Ville distribution and the Wavelets Transform are useful techniques to detect patterns in signals, even on non stationary signals.

The wavelet transform has been used for analysis of non stationary signals because this is a linear and invertible transform that uses a vectorial basis, generated from functions that have both time and frequency fixed localization. Mallat and Zhang (1993) proposed an algorithm denoted "Matching Pursuit" capable to detect wave patterns in the signals representing it through wavelets.

This work propose a methodology for the identification of mechanical systems modal parameters of mechanical systems through wavelets, using a modification of an algorithm developed by Mallat and Zhang (1993), including optimization techniques to determine the wavelets of a family that are present in the time response signal.

The proposed method is compared with the one of Ruzzene et al (1997), which detects the envelope through wavelet transform. His method is a similar procedure that detects signal envelopes using the Hilbert transform.

To evaluate the robustness and the accuracy of the proposed method, signals generated in the simulation of a three degree of freedom vibratory systems and as well as signals obtained from experimental apparatus are used.

2. WAVELETS : MATHEMATICAL BACKGROUND

Wavelets are functions obtained by translation and dilation of a function called mother wavelet. The dilation defines the mother wavelet central value position in the frequency domain and the translation define the central value in time domain. Both operation are realized by two parameters *a* and *b* in the following equation :

$$\psi_{a,b}(t)=\frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) \quad , \quad a>0 \quad , \quad b\in R \tag{1}$$

The translation and dilation occur if the function Ψ has the following properties:

$$|\psi(t)| \leq c(1+|t|)^{-1-\varepsilon} \qquad and \qquad |\psi(\omega)| \leq c(1+|\omega|)^{-1-\varepsilon} \quad with \quad \varepsilon>0 \tag{2}$$

The above conditions are sufficient for existence of both central values in the frequency and time domains.

The function that define the mother wavelet should have finite norm and zero mean, as shown in "Eq.(3)" :

$$\|\psi(t)\|^2 = \int_{-\infty}^{+\infty} |\psi(t)|^2 dt < \infty \qquad and \qquad C_\psi = \int_{-\infty}^{+\infty} \frac{\psi(\omega)}{|\omega|} d\omega < \infty \tag{3}$$

These restrictions are sufficient for the existence of the wavelet transform, once it is defined as the scalar product of the wavelet with signal *f(t)* as follows:

$$TW(a,b)=\langle f,\psi_{a,b}\rangle=\frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty} f(t)*\psi\left(\frac{t-b}{a}\right)dt \tag{4}$$

The inverse transform is given by:

$$f(t)=C_\psi^{-1}\int_0^\infty \frac{da}{a}\int_{-\infty}^{+\infty} TW(a,b)\psi_{a,b}(t)db \tag{5}$$

To guarantee mother wavelet's oscillatory characteristic, with capacity to detect the variations in the signals shape, Kovacevic & Cohen (1996) propose the following mathematical cancellations set:

$$\left(\left(\frac{\partial}{\partial\omega}\right)^k \hat{\psi}\right)(0)=0 \qquad and \qquad \int_{-\infty}^{\infty} t^k\psi(t)dt=0, \quad k=0,1,\ldots,N \tag{6}$$

These conditions are sufficient for the existence of wavelet transform. However, for the division of a signal in a wavelet series it is necessary the use of frames and a linear operator. Vectorial spaces generated by a wavelet's family must be orthogonal and orthonormal. (Kovacevic & Cohen, 1996)

These restrictions intend to guarantee the representation of a discrete signal by a finite group of sequences, as the defined in "Eq.(7)". The first term is the scalar product of two vectors, as defined by "Eq.(4)".

$$\langle f, \psi_{a,b} \rangle \psi_{a,b}(t) \tag{7}$$

If these orthogonality and orthonormality conditions are satisfied, the signal *f* can be represented by the following summation :

$$f = \sum \langle f, \psi_{a,b} \rangle \psi_{a,b} \tag{8}$$

Kovacevic & Cohen (1996) proposed that the orthogonality condition is obtained when a symmetrical and orthonormal window is multiplied by orthogonal functions. This is always fulfilled by using trigonometric functions.

This condition is necessary when the subsets formed by vectorial spaces possess a non null intersection. Therefore, when these intersection are null the subsets are always orthogonal. The subsets orthogonality guarantees that total set can be formed by direct summation.

3. MODIFIED "MATCHING PURSUIT" ALGORITHM

Mallat & Zhang (1993) proposed an algorithm to identify the components of a discrete signal that has high correlation coefficients, with the vectors Y that are members of a set called dictionary, as defined in the "Eq.(9)", where Γ is the set of wavelets parameter.

$$D = (\psi_{?})_{?\in G} \qquad \text{and} \qquad \|\psi\| = 1 \tag{9}$$

By definition this dictionary is said complete only if it is a Hilbert space. Thus, the representation of a signal belonging to the vectorial Hilbert space *(H)* is done through successive approaches as follows:

$$f = \langle f, \psi_{\gamma_0} \rangle \psi_{\gamma_0} + R_f \tag{10}$$

The term R_f is called orthogonal residue of *f* in relation to Ψ . This statement is true since the following relationship is verified :

$$\|f\|^2 = |\langle f, \psi_{\gamma_0} \rangle|^2 + \|R_f\|^2 \tag{11}$$

The approximation of f will be better when the norm of the residue is minimum. To minimize this residue we can find the wavelet from the dictionary that maximize the factor k, normalized between zero and the unity.

$$k = \frac{\left|\left\langle f, \psi_{\gamma_0} \right\rangle\right|}{\|f\|} \tag{12}$$

This factor assume the unitary value only if a dictionary's component is capable to represent the signal in its totality. The maximum value of k decreases when the relationship of the RMS level of some strange patterns to the dictionary and the dictionari's patterns is increased.

Once the residue is determined in the first interaction, defined in "Eq.(10)", the algorithm continues by substituting the initial vector by the residue. The interactions series in "Eq.(13)" can be written, where it should be noticed that for $n=0$, R_f^0 is the original vector f.

$$R_f^{\ n} = \left\langle R_f^{\ n}, \psi_{\gamma_n} \right\rangle \psi_{\gamma_n} + R_f^{\ n+1} \quad n = 0, \ldots, m \tag{13}$$

Consequently the signal can be reconstructed by the following summation:

$$f = \sum_{n=0}^{m-1} \left\langle R_f^{\ n}, \psi_{\gamma_n} \right\rangle \psi_{\gamma_n} + R_f^{\ n+1} \tag{14}$$

The convergence of the algorithm is demonstrated by Mallat & Zhang (1993) using the signal energy conservation method or applying the concept of orthogonality.

This algorithm was developed initially for dictionaries formed only by wavelets with two dimensional resolutions. In this case the wavelets parameters are obtained through the solution of a linear equations set.

In this work the "Simulated Annealing" and conventional optimization techniques are combined to modify the Mallat's algorithm. This procedures is used to identify the wavelet of a dictionary that maximize the factor k.

The optimization techniques must be used because the number of wavelets parameters is greater than two. This metodology leads to a significant reduction in the number of wavelets parameters combinations to achieve the final solution.

The algorithm convergence rate decrease exponentially, and its decay depends on the residue and dictionary elements correlation level. (Mallat and Zhang, 1993)

For signals components which have high levels RMS compared to background noise, the norm residue decay rate will be high.

When all signals patterns are identified, the residue will not have correlation with the dictionary. It behaves as in the analysis of a white noise.

When the residue correlation level approaches the white noise correlation level with the same dictionary, the algorithm may be stopped. This condition is expressed by "Eq.(15)", where the term $k(R_w)$ represents the white noise residue correlation level.

$$k\left(R_f^{\ n}\right) > E\left(k\left(R_W\right)\right) \qquad \text{for} \qquad 0 \le n \le m \tag{15}$$

4. APPLICATION IN MODAL ANALYSIS

In order to apply the proposed method to modal parameter identification it is necessary define a wavelet capable to generate a dictionary correlated with the patterns found in the Impulse Response Function of a mechanical systems.

Freudinger et al (1998) used the Laplace's Wavelet to characterize some aircraft modal parameters in a real time during flight. This wavelet is not orthonormal, leading to inadequate results.

The impulse response function of a multi-degree of freedom vibratory systems expressed in modal coordinates in "Eq.(16)", is used to build the wavelet dictionary.

$$\{x\}=[u]\{\eta\} \quad , \quad \eta_r = -C_1 * u_j^r e^{-\frac{?}{\sqrt{1-?^2}}?_r t} \cos(\omega_r t + \phi) \tag{16}$$

The mother wavelet is presented in "Eq.(17)", where the denominator is the wavelet norm.

$$\psi_{\xi,\omega,\phi} = \frac{e^{-\frac{?}{\sqrt{1-?^2}}?(t)} \cos(\omega t + \phi)}{\sqrt{\int_0^T \left(e^{-\frac{?}{\sqrt{1-?^2}}?(t)} \cos(\omega t + \phi) \right)^2 dt}} \tag{17}$$

For this application, the wavelet translation in time domain does not have to be performed by the algorithm, since all IRF components start at the time origin and decrease exponentially as function of the modal parameters.

Substituting this wavelet in the "Eq(13)" it can be verified that the inner product value is the product of the initial conditions by the eigenvectors and that ϕ is the eigenvectors phase angle.

The algorithm defined by "Eq.(13)" will identify the modal parameters, choosing the higher correlated wavelets that are contained in the dictionary.

5. SIMULATED AND EXPERIMENTAL RESULTS

The mechanical system model that represents the apparatus used in experimental tests is shown in "Fig.1". The model values of mass, stiffness and damping are respectively M_1=3.5 Kg, M_2=2 Kg, M_3=1.5 Kg, K_1=5 10^5 N/m, K_2=2 10^5 N/m, K_3=1 10^5 N/m, C_1=100 N/(m/s).

The time responses used in all simulation tests have 2048 data points, spaced by 39 ms, resulting a frequency resolution equal to 0.125 Hz.

The robustness and precision of the proposed method are verified by the analysis of the IRF obtained from theoretical model adding several noise levels to the simulated response. The results obtained are shown in "Table 1" and in "Fig.2". The behavior of correlation indexes, that determines the stop condition for the algorithm is presented in "Fig.3".

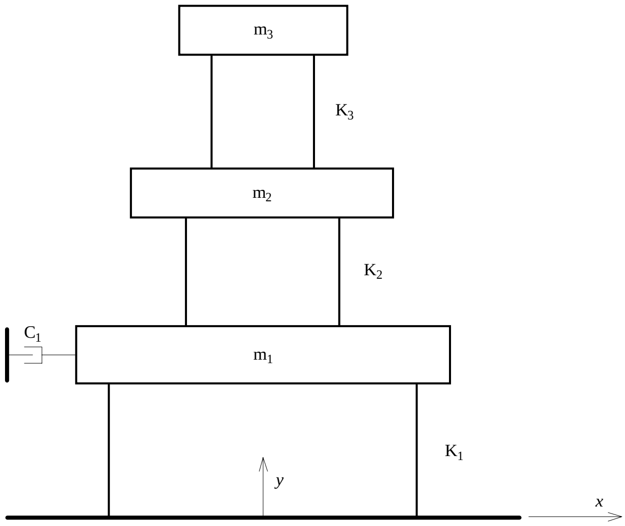

Figure 1- Theoretical model and experimental apparatus schematic diagram.

The inclusion of optimization techniques in the algorithm permits to obtain parameters values that are very close to their global optimum value. This guarantees the method robustness in the presence of up to 20 % noise levels, allowing a good signal representation, as presented in “Fig.2”.

Table 1- Theoretical model: Values of the real and identified modal parameters.

Modes	Identified Values				
	Noise Level	Estimated Frequency	Difference	Estimated Damping Factor	Difference
Mode 1	Null	27.0287	0.15%	0.0046	2.12%
Natural Frequency [Hz]	1%	27.0295	0.16%	0.0045	4.25%
26.9858	5%	27.0270	0.15%	0.0046	2.12%
Damping Factor	10%	27.0265	0.15%	0.0044	6.81%
0.0047	20%	27.0311	0.16%	0.0047	zero
Mode 2	Null	56.8451	0.17%	0.0167	15.27%
Natural Frequency [Hz]	1%	56.8345	0.15%	0.0145	0.69%
56.7477	5%	56.8320	0.15%	0.0143	0.69%
Damping Factor	10%	56.8459	0.17%	0.0142	1.38%
0.0144	20%	56.8485	0.17%	0.0166	13%
Mode 3	Null	81.3843	0.17%	0.0166	1.83%
Natural Frequency [Hz]	1%	81.3627	0.02%	0.0200	18%
81.2424	5%	81.3461	0.13%	0.0199	17.59%
Damping Factor	10%	81.3759	0.16%	0.0186	11.827%
0.0164	20%	81.3449	0.12%	0.0165	0.6%

In “Table 1” it is noticed that all vibration modes estimated frequencies presented differences lower than 0.17% of the correct values. However, the estimated damping factors have differences up 18%, because the objective function is ill conditioned to this design variable, independently of the added noise level.

The random behavior of the damping factor differences may be explained by the randon nature of the “Simulated Annealing” optimization method, which always gives values close to the global optimum.

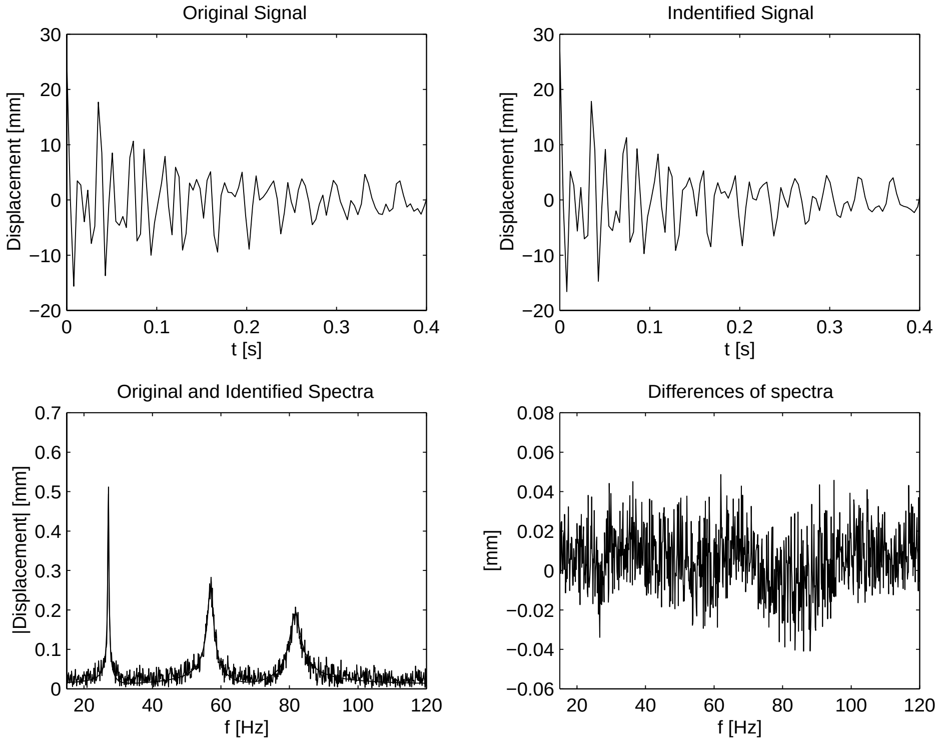

Figure 2- Effect of 20% noise level added to the simulated Impulse Response.

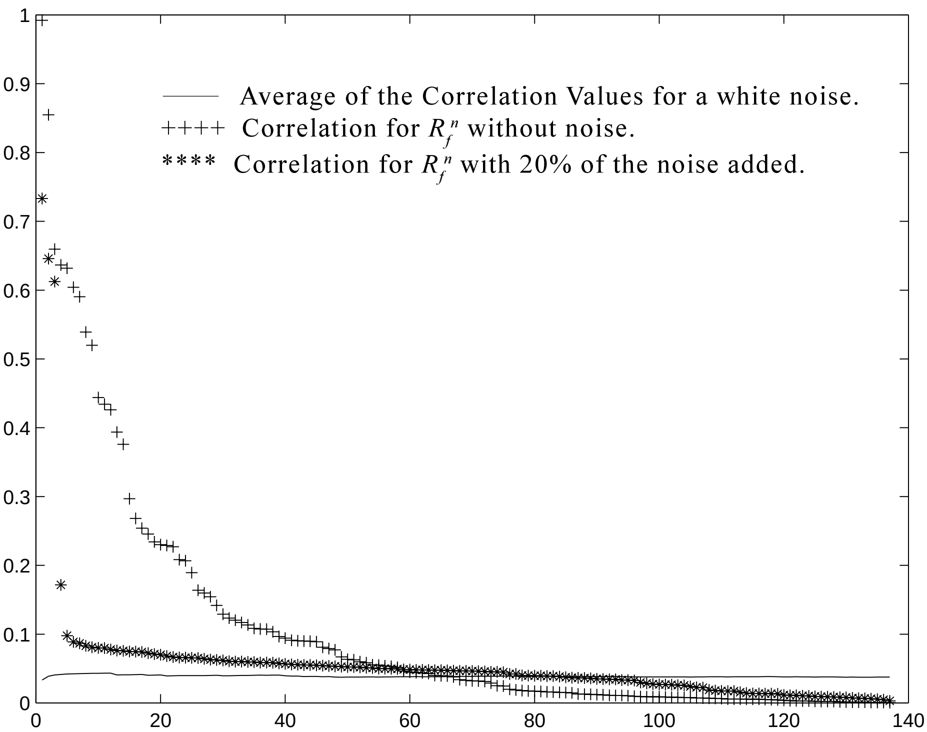

Figure 3- Noise level effect on the behavior of the Correlation coefficients (k) as function of iteration number.

As can be seem in “Fig. 3”, the correlation level *k* decreases rapidly when the modal components are extracted from the residue, even for high added noise levels. It is noticed that the algorithm stop criterion is achieved around 70 interactions, independently on the presence of background noise, where the $k(R_f^n)$ curves crosses the white noise correlation coefficient mean value curve

To evaluate the algorithm capacity to identify the mechanical modal parameters for systems with high modal density, the values for mass, stiffness and damping of model were altered to: $M_1=3.3$ *Kg*, $M_2=2.223$ *Kg*, $M_3=0.929$ *Kg*, $K_1=9.6052\ 10^5$ *N/m*, $K_2=5.3982\ 10^4$ *N/m*, $K_3=1.9706\ 10^5$ *N/m*, $C_1=200$ *N/(m/s)*, resulting a difference of 0.2474 Hz between the second and the third natural frequencies, which corresponds to two resolution lines separation in frequency domain.

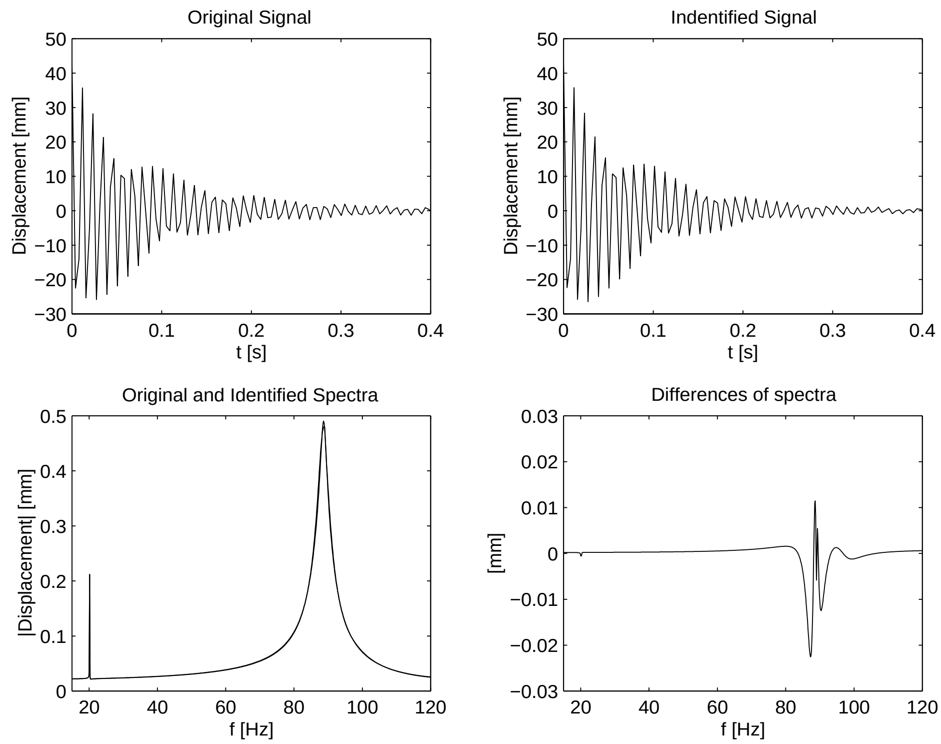

Figure 4- Impulse response for a mechanical system with high modal density.

The estimated modal parameters are presented in “Table 2.”. It can be noticed that the frequency differences are lower than 0.22%.

The wavelets that correlates to the second and third modes are very similar. After the first wavelet extraction, a small error in its parameters causes a high error in the next wavelet identification. These amplitude error of the wavelets leads to a significant error in the modal damping factor estimation. It should be noticed that this is a critical case for modal analysis, independently of the applied method.

The correct and estimated spectra difference in “Fig.4” is high only at frequencies close to the second and third modes.

The experimental tests were conducted on the vibratory system shown in “Fig.1”. The impulsive excitation is applied at mass 1 by a B&K impact hammer equipped with a piezoelectric force transducer. The response is measured by a B&K accelerometer installed at mass 1. The time signals are digitalized by a Spectral Dynamics analyzer with 2048 point and 1.95 ms time resolution.

The synchronized time average of 10 samples, stored on analyzer internal memory, is transfered to a microcomputer. The force signal is used to trigger the acquisition and to synchronize the time average process.

The estimated modal parameters of the experimental vibratory system are presented in "Table 3". These results were obtained by the algorithm, and represent the mean and variance of 20 experiments. The statistical confidence is assured by very low values of the variances.

Table 2. Real values and obtained for a mechanical systems with high modal density.

Modes	Identified Values			
	Estimated Frequency	Difference	Estimated Damping Factor	Difference
Mode 1 Natural Frequency: 20 [Hz] Damping Factor: 0.0006	20.0320	0.16%	0.0006	zero
Mode 2 Natural Frequency: 88.2716 [Hz] Damping Factor: 0.0290	88.4962	0.22%	0.0209	28%
Mode 3 Natural Frequency: 88.4195 [Hz] Damping Factor: 0.0118	88.5778	0.17%	0.002	83%

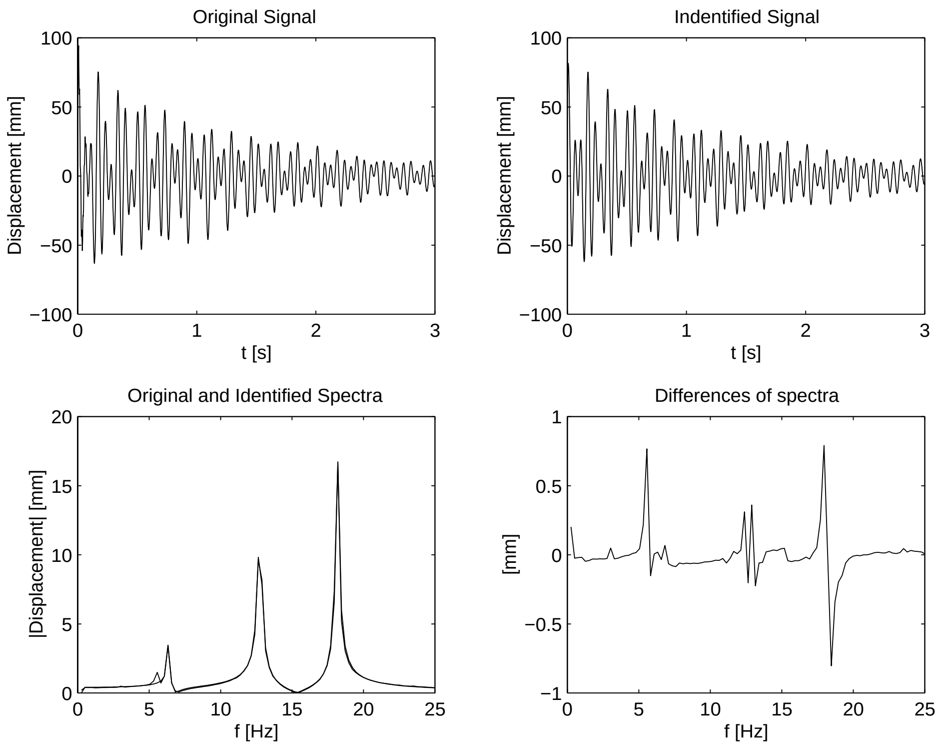

Figure 5- Impulse response and obtained by algorithm to a mechanical systems tested in the laboratory.

Table 3. Experimental estimated modal parameters mean values and variance.

Identified Modal Parameters				
Mode	Frequency [Hz]	Variance	Damping	Variance
1º	6.117	$7.32\ 10^{-5}$	0.00871	$2.339\ 10^{-6}$
2º	12.484	$8.944\ 10^{-5}$	0.00893	$1.02\ 10^{-7}$
3º	17.92078	$3.2\ 10^{-4}$	0.00535	$1.6\ 10^{-8}$

6. CONCLUSIONS

Good signal representation and modal parameters extraction are feasible by the inclusion of the “Simulated Annealing” optimization technique in the “Matching Pursuit” algorithm. The proposed method is insensitive to the objective function ill conditioning which guarantees its robustness, being responsible to the reduction of the computational effort.

The proposed metodology has better performance than that of Ruzzene et al (1996) method, because the precision and robustness of the modal parameter identification of the proposed method is independent of wavelet initial parameters. The Ruzzene’s method requires the prior knowledge of the approximate natural frequencies values for a given vibrating system.

In the case of high modal density systems with small modal damping ratios, the obtained results are better than those calculated by the methods based on the FRF estimation, which demand a high frequency resolution.

7. ACKNOWLEDGMENTS

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